LIST OF ENTRANCE EXAM QUESTIONS

FOR THE INTERNATIONAL MASTER'S DEGREE PROGRAM

ITMO

ADVANCED QUANTUM AND NANOPHOTONIC SYSTEMS

"Nanophotonics" track

1. The law of conservation of electric charge. Coulomb's law. Electric field strength. Superposition principle. Model of continuous charge distribution. Electric field strength of a uniformly charged ring and filament.
2. Flux of electric field strength vector. Gauss's theorem for electrostatic field strength vector. Application of the Gauss's theorem for a point charge and a plane.
3. Electric field potential. Electric potential of a point charge. Relationship between the electrostatic field vector and the potential. Poisson's equation. Potential of a uniformly charged sphere.
4. Electric dipole. Field strength and electrostatic potential of a point dipole. Electric dipole in an external electric field (force, torque, potential energy).
5. The concept of electrical capacitance. Examples of capacitors with different geometric configurations. Derivation of the capacitance of a parallel plate capacitor.
6. Magnetic field B vector. Magnetic field of a conductor with current, Biot-Savart-Laplace law. Magnetic field of a finite-length straight conductor with direct current.
7. Circulation theorem for magnetic field vector. Magnetic field in the center of a ring with direct current. Expression for the magnetic field in a long solenoid. Inductance.
8. Electromotive force. Generalized Ohm's law (differential and integral forms). Power in a DC circuit. Joule-Lenz law (differential and integral forms).
9. Electromagnetic field. Maxwell's equations in integral and differential forms, their physical meaning. Basic electromagnetic quantities and laws in different unit systems: SI, CGS, and Gaussian.
10. Derivation of electromagnetic plane wave equation from Maxwell's equations. Transverse nature of electromagnetic plane wave, relation between electric and magnetic fields, in-phase oscillations of electric and magnetic fields.
11. Polarization states of a plane harmonic wave. Elliptical, circular, and linear polarization. Polarized and natural light, Malus law, degree of polarization.
12. Diffraction of light. The Huygens-Fresnel principle: definition and mathematical formulation. Fresnel spiral, Fresnel zone plate.
13. Diffraction by a circular hole and a circular screen (Fresnel zones, Fresnel spiral)
14. Diffraction at a rectilinear edge of an opaque screen. Cornu spiral.
15. Fraunhofer diffraction by a slit. Properties of the diffraction pattern.
16. Interference of light. Conditions for the interference formation, basic relations and characteristics of the interference field. Types of interference fringes.
17. Refraction of electromagnetic waves. Derivation of Snell's law. Total internal reflection.
18. Polarization of electromagnetic waves during reflection and refraction. Fresnel formulas. Electromagnetic surface waves. Application of Fresnel formulas: Brewster's law. Phase relations for electromagnetic waves at the boundary of two media.
19. Dispersion of light. Frequency and spatial dispersion. Electronic theory of frequency dispersion. Permittivity-frequency dependence.
20. Propagation of an electromagnetic wave packet in a dispersive medium. Group velocity. Rayleigh formula.
21. Nonlinear polarization of a medium. Nonlinear optical phenomena (harmonic generation, addition and subtraction of frequencies, self-focusing, stimulated scattering).
22. Features of electromagnetic waves' propagation in dielectric waveguides.
23. Optical planar waveguides. Introduction to waveguide modes.
24. Optical fiber. Fiber structure. Propagation of light in a fiber.
25. Classification (types) of lasers. Characteristic features of lasers of various types. Main characteristics of laser radiation and methods for their evaluation.
26. Absorption and generation of optical radiation in semiconductors. Light-emitting diodes. Design and operation of the simplest semiconductor laser.
27. Photonic crystals. The use of photonic crystals for information transmission, storage and processing. Formation of band structure in photonic crystals.

28. Postulates of quantum mechanics: the uncertainty principle, Schrödinger equation, wave function of the system, basic quantum mechanical operators and commutation relations between them.
29. The principle of identity of particles. Permutation symmetry. Second quantization: creation and annihilation operators, commutation relations for Bose and Fermi statistics.
30. Two-level system in a resonant electromagnetic field. Rabi oscillations.
31. Mechanical and magnetic moments of electron. Spin. Pauli matrices. Commutation relations
32. Solution of the Schrödinger equation for a hydrogen-like atom.
33. Stationary perturbation theory. Application for the case of a degenerate energy level.
34. Non-stationary perturbation theory. Transitions induced by a finite-time perturbation.
35. Stationary states of a particle in the semiclassical case. Bohr-Sommerfeld quantization rule.
36. Born approximation in scattering theory.

"Quantum materials" track

1. The principle of least action. Euler-Lagrange equations of motion. Hamilton's canonical equation.
2. Postulates of quantum mechanics. The harmonic oscillator problem in classical mechanics. One-dimensional anharmonic oscillator.
3. Schrödinger equation. Schrödinger equation in an infinite rectangular potential well. Schrödinger equation in periodic systems.
4. Commutator. Heisenberg equation. The uncertainty principle.
5. Schrödinger equation for multilayer structures. Transfer matrix method. Schrödinger equation in periodic systems.
6. Harmonic oscillator. Eigen energies and functions.
7. Coherent states.
8. Angular momentum operator. Motion in centrally symmetric field. Spin. Spin operator, uncertainty relations for spin and angular momentum. Additions of angular momenta.
9. The principle of indistinguishability of particles.
10. Bose statistics. Bose condensation
11. Fermi statistics. Pauli principle.
12. Perturbation theory. Time-independent perturbations. Time-dependent perturbations. Non-resonant case. Time-dependent perturbations. Resonant case.
13. Bloch's theorem. Band diagram of crystals.
14. Poynting vector and density of electromagnetic energy. Continuity equation.
15. Propagation of waves in anisotropic media. Refraction of light at a boundary between vacuum and anisotropic medium. Birefringence.
16. Fabry-Perot resonator. Eigen frequencies and quality factor of the resonator.
17. Photonic crystals. Photonic bandgap. Dispersion relation for a one-dimensional photonic crystal.
18. Gauge invariance of Maxwell's equations. Relation between gauge invariance in quantum mechanics and electrodynamics.
19. Dyadic Green's function. Dyadic Green's function of an electric dipole in vacuum.
20. Electromagnetic field in vacuum. Physical meaning and content of Maxwell's equations. Plane waves. Derivation of electromagnetic plane wave equation from Maxwell's equations. Transverse nature of electromagnetic plane wave, relation between electric and magnetic fields, in-phase oscillations of electric and magnetic fields.
21. Electromagnetic field in a continuous medium. Maxwell's equations for continuous media. Polarization of a medium, displacement currents, permittivity, refractive index. Method of images.
22. Reflection and refraction of an electromagnetic wave at the boundary of two homogeneous media. Fresnel formulas. Electromagnetic surface waves.
23. Brewster's law. Phase relations at the boundary of two media. Polarization states of a plane wave. Elliptical, circular, and linear polarization. The concepts of anisotropy and birefringence.
24. Interference and diffraction of light. The simplest interference and diffraction devices. Huygens principle. Diffraction patterns. Diffraction by a circular hole, a circular screen, and at a rectilinear edge of an opaque screen.
25. Dispersion of light. Frequency and spatial dispersion. Phenomenological theory of frequency dispersion. General form of the frequency dependence of the refractive index. Transparency bands and fundamental absorption bands, their mutual position. Normal and anomalous dispersion.

“Hybrid materials” track

1. Laws of geometric optics.
2. Electromagnetic field in vacuum. Physical meaning and content of Maxwell's equations. Plane waves. Derivation of electromagnetic plane wave equation from Maxwell's equations. Transverse nature of electromagnetic plane wave, relation between electric and magnetic fields, in-phase oscillations of electric and magnetic fields.
3. Electromagnetic field in a continuous medium. Maxwell's equations for continuous media. Polarization of a medium, displacement currents, permittivity, refractive index. Method of images.
4. Propagation of waves in anisotropic media. Refraction of light at a boundary between vacuum and anisotropic medium. Birefringence.
5. Reflection and refraction of an electromagnetic wave at the boundary of two homogeneous media. Fresnel formulas. Electromagnetic surface waves.
6. Brewster's law. Phase relations at the boundary of two media. Polarization states of a plane wave. Elliptical, circular, and linear polarization. The concepts of anisotropy and birefringence.
7. Interference and diffraction of light. The simplest interference and diffraction devices. Huygens principle. Diffraction patterns. Diffraction by a circular hole, a circular screen, and at a rectilinear edge of an opaque screen.
8. Dispersion of light. Frequency and spatial dispersion. Phenomenological theory of frequency dispersion. General form of the frequency dependence of the refractive index. Transparency bands and fundamental absorption bands, their mutual position. Normal and anomalous dispersion.
9. Postulates of quantum mechanics. The harmonic oscillator problem in classical mechanics. One-dimensional anharmonic oscillator.
10. The principle of indistinguishability of particles.
11. Bloch's theorem. Band diagram of crystals.
12. Band theory of solids
13. X-ray diffraction. Method of crystal structure analysis.
14. Basic concepts of crystal lattice symmetry. Miller index. Symmetry point group.
15. Types of defects in a solid.
16. Chemical bond main types. Characteristics of a chemical bond in molecules: energy, length, valence angle, order (multiplicity) and polarity. The concept of hybridization of atomic orbitals. Basic principles of the molecular orbital method (MO theory). MO LCAO method (linear combination of atomic orbitals).
17. Basic concepts of chemistry of complex compounds: central atom and its coordination number; ligands, denticity, donor atom, inner and outer coordination spheres. Isomerism of complex compounds. The concept of the classification of complex compounds. Chelation effect.
18. Disperse systems. Surface tension. Surfactants, their effect on surface tension. Gibbs adsorption equation.
19. Micellization in aqueous and non-aqueous media. Methods for obtaining dispersed systems and their stabilization factors.
20. Main functional groups and classes of organic compounds. Types of isomerism of organic compounds. The concept of conformations on the example of alkanes. Geometric isomerism of alkenes. The concept of enantiomers and racemates. R,S-nomenclature. Compounds with two chiral centers. The concept of diastereomers.

“Computer modeling of quantum and nanophotonic systems” track

1. Linear space, basis and dimension. Linear transformation of finite-dimensional spaces, its matrix. Kernel and image of linear transformation. Eigenvalues and eigenvectors of linear transformations. Diagonalizable of linear transformations.
2. Self-adjoint transformations of Euclidean spaces, properties of their eigenvalues and eigenvectors. Reduction of quadratic forms in linear space to canonical form. Positive definite quadratic forms. Sylvester's criterion.
3. Groups, group order. Homomorphism and isomorphism of groups. Direct product of groups. Cyclic groups.
4. Linear ordinary differential equations with constant coefficients and quasi-polynomial right-hand side. Systems of linear homogeneous differential equations with constant coefficients, methods for their solution.
5. Linear ordinary differential equations with variable coefficients. Fundamental solution system. Wronskian determinant. Liouville-Ostrogradski formula.

6. Problems of the calculus of variations. A functional. Variation of functional. Necessary conditions for a local extremum. The main lemma of the calculus of variations.
7. Differentiability of functions of a complex variable. Cauchy-Riemann conditions. Cauchy integral theorem. Cauchy's integral formula. Taylor series expansion of a function that is regular in the vicinity of a point.
8. Laurent series expansion of a function regular in a ring. Isolated singular points of a single-valued nature. Residues. Calculation of integrals over a closed contour using residues.
9. Entire functions and Liouville's theorem. Meromorphic functions and the Mittag-Leffler theorem.
10. Machine numbers and machine arithmetic. Sources and classification of errors. Elementary theory of errors.
11. Numerical differentiation. Lagrange and Newton interpolating polynomials. Numerical integration. Estimation of errors. Gauss quadrature formulas.
12. The concept of numerical methods for solving ordinary differential equations. Approximation, stability, convergence. Euler method. Runge–Kutta methods.
13. The concept of difference methods for solving partial differential equations. Approximation, stability, convergence. Initial and boundary conditions. Solution of a one-dimensional hyperbolic equation, stability of the scheme.
14. Linear oscillations of a point mass: free undamped, free damped, and forced oscillations. Q factor. Resonance and resonance curve.
15. Waves. Wave equation. Waves in one-dimensional chains of coupled oscillators. Dispersion equation. Wave packet, phase and group velocities in optics and quantum mechanics.
16. Electromagnetic field. Maxwell's equations for continuous media. Physical meaning and content of equations. Polarization of a medium, displacement currents, permittivity, refractive index.
17. Energy density of electromagnetic field in vacuum and in matter. Poynting's theorem.
18. Electromagnetic waves. Wave vector. Polarization. Standing waves. Plane waves. Fresnel formulas.
19. Main principles of quantum mechanics. Stationary and non-stationary Schrödinger equations. Physical quantity. Operators of basic physical quantities.
20. Diffraction and interference on the example of waves and particles. De Broglie hypothesis.

RECOMMENDED READING

1. L.D. Landau, E.M. Lifshitz. Mechanics (Volume 1 in Course of Theoretical Physics).
2. L.D. Landau, E.M. Lifshitz. Electrodynamics of Continuous Media. (Volume 8 in Course of Theoretical Physics).
3. Cohen-Tannoudji. Quantum mechanics (in two volumes).
4. Sakurai. Modern quantum mechanics.
5. N. I. Kaliteevsky. Wave optics.
6. J. Jackson. Classical electrodynamics.
7. L.D. Landau, E.M. Lifshitz. The Classical Theory of Fields (Volume 2 in Course of Theoretical Physics, any edition).
8. M. Born, E. Wolf. Principles of Optics.
9. Bohm. "Quantum Theory".
9. O.A. Reutov, A.L. Kurtz, K.P. Butin. Organic chemistry (in 4 volumes).
10. N.L. Glinka. General Chemistry.
11. M. Smith. March's Organic Chemistry. Volume 1-4. 2020.